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Electro-magneto-thermo-visco-elastic Plane Waves in Rotating Media with Thermal Relaxation

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A study is made of the propagation of plane electro-magneto-thermo-visco-elastic harmonic waves in an unbounded isotropic conducting visco-elastic medium of Kelvin–Voigt type permeated by a primary uniform magnetic field when the entire medium rotates with a uniform angular velocity. The thermal relaxation time of heat conduction, the electric displacement current, the coupling between heat flow density and current density, and that between the temperature gradient and the electric current are included in the analysis. A more general dispersion relation is obtained to determine the effects of rotation, thermal relaxation time, visco-elastic parameters, and the external magnetic field on the phase velocity of the waves. Perturbation techniques are used to study the influence of small magneto-elastic and thermo-elastic couplings on the phase velocity of the waves. Cases of low and high frequencies are also analyzed to determine the effect of rotation, visco-elastic parameters, thermo elastic and magneto-elastic coupling, as well as thermal relaxation time of heat conduction on the waves.

KEY WORDS: magneto-thermo-visco-elastic waves; rotating medium; thermal relaxation.

1. INTRODUCTION

The propagation of electro-magneto-thermo elastic waves in an electrically and thermally conducting unbounded isotropic non-rotating media has been considered by many authors [1–7]. Paria [1,2] used the classical Fourier's law of heat conduction and neglected the electric displacement current, the coupling between the current density, and the heat

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flow density, that between the temperature gradient and the current density. Nayfeh and Nemat-Nasser [8] used a more general model of electro-magneto-thermo-elasticity, which includes the thermal relaxation time of heat conduction and the electric displacement current and studied the propagation of plane waves in a non-rotating medium. Schoenberg and Censor [9] studied the propagation of plane harmonic waves in a rotating elastic medium where it is shown that the elastic medium becomes dispersive and anisotropic due to rotation.

It appears from the above discussion that little attention has been given to the propagation of thermo-elastic plane waves in a rotating medium in the presence of an external magnetic field using a more general model of electro-magneto-thermo-elasticity. Since most large bodies like the earth, the moon, and other planets have an angular velocity, it would appear more realistic, on the considerations of the visco-elastic nature of the earth [10, 11], to consider the propagation of magneto-thermo-elastic plane waves in a rotating visco-elastic medium. It is very likely that rotation and viscoelastic parameters will have some important effects on coupled magneto-thermo-visco-elastic plane waves. Roy Choudhuri and Debnath [12] considered a problem of plane wave propagation in a rotating medium, but neglected the displacement current, charge density, the coupling between heat flow and the current density, and that between the temperature gradient and the current density.

The main object of this paper is to study the propagation of plane harmonic waves in an infinite conducting thermo-visco-elastic medium (Kelvin-Voigt type) permeated by a primary uniform magnetic field when the entire medium rotates with a uniform angular velocity, using a more general model of electro-magneto-thermo-visco-elasticity by including (i) the displacement current, (ii) the thermal relaxation time of heat conduction (Lord-Shulman's theory], (iii) the coupling between the heat flow and the current density, and (iv) the coupling between the temperature gradient and the current density.

A more general dispersion relation is obtained to determine the effects of rotation, thermal relaxation time, the external magnetic field, and visco-elastic parameters on the phase velocity of the waves. Special attention is given to the effects of rotation, relaxation time, visco-elastic parameters, and small thermo-elastic as well as small magneto-elastic couplings on the phase velocity of the waves. It may be mentioned that a similar problem in rotating medium without visco-elastic effects was studied by Roy Choudhuri and Debnath [12], and Roy Choudhuri [13], by Bakshi et al. [14], and by Othman [15] in generalized thermoelasticity.

2. FORMULATION OF THE PROBLEM AND THE BASIC EQUATIONS

We considered an unbound, isotropic, thermally, and electrically conducting visco-elastic medium (Kelvin-Voigt type) permeated by a primary uniform magnetic field to be $\underline{H_0}$. The medium is characterized by the density ρ, Lame' constants λ, μ, and visco-elastic parameters λ^1, μ^1 and is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega \underline{k}$ where $\underline{k}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in a rotating frame of reference is

$$\rho\,[\underline{\ddot{u}} + \underline{\Omega} \times (\underline{\Omega} \times \underline{u}) + 2\underline{\Omega} \times \underline{\dot{u}}]$$

$$[(\lambda+\mu) + (\lambda^1+\mu^1)\tfrac{\partial}{\partial t}]\underline{\nabla}(\underline{\nabla} \cdot \underline{u}) + (\mu + \mu^1 \tfrac{\partial}{\partial t})\nabla^2 \underline{u} + \underline{j} \times \underline{B} - \gamma_0 \text{ grad}\,\theta \quad (1)$$

Maxwell's equations are

$$\text{curl } \underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t}, \quad (2)$$

$$\text{curl } \underline{E} = -\frac{\partial \underline{B}}{\partial t}, \quad (3)$$

$$\text{div}\,\underline{B} = 0, \qquad \text{div } \underline{D} = \rho_e, \quad (4)$$

$$\underline{B} = \mu_e \underline{H}, \quad \underline{D} = \in \underline{E}. \quad (5)$$

The generalized Ohm's law with the effect of the temperature gradient on current is

$$\underline{j} = \sigma[\underline{E} + \left(\frac{\partial u}{\partial t} + \underline{\Omega} \times \underline{u}\right) \times \underline{B}] - k_0 \text{ grad}\,\theta. \quad (6)$$

The local energy balance principle yields. $-h_{i,i} = \rho c_v \dot{\theta} + \gamma_0 \theta_0 \dot{\Delta}$ (7)

The generalized Fourier's law of heat conduction [10] is

$$\tau_0 \dot{h}_i + h_i = -k\theta_{,i} + \Pi_0 j_i, \quad i = 1, 2, 3. \quad (8)$$

All symbols have their usual meanings as in Ref. 8.

Equations (7) and (8), on elimination of h_i, give the generalized heat conduction equation (Lord–Shulmon's model of generalized thermoelasticity [17]):

$$\rho c_V (\dot{\theta} + \tau_0 \ddot{\theta}) + \gamma_0 \theta_0 (\dot{\Delta} + \tau_0 \ddot{\Delta}) = k\nabla^2 \theta - \Pi_0 j_{i,i}. \quad (9)$$

Eliminating $\underline{j}, \rho_e, \underline{D}, \underline{B}$ in Eqs. (1)–(6) and (9), linearizing by setting $\underline{H} = \underline{H_0} + \underline{h^*}$, where $\underline{h^*}$ is the perturbed field of $\underline{H_0}$, and assuming that the products of $\underline{h^*}, \underline{E}, \underline{u}, \theta$ and their derivatives to be very small, the non-linear coupled partial differential equations reduce to

$$\rho c_V(\dot{\theta} + \tau_0\ddot{\theta}) + \gamma_0\theta_0(\dot{\Delta} + \tau_0\ddot{\Delta}) = k\nabla^2\theta + \Pi_0(\in \underline{\nabla} \cdot \underline{\dot{E}}), \tag{10}$$

$$\begin{aligned} \rho[\underline{\ddot{u}} + \underline{\Omega} \times (\underline{\Omega} \times \underline{u}) + 2\underline{\Omega} \times \underline{\dot{u}}] = (\lambda + \mu)\underline{\nabla}(\underline{\nabla} \cdot \underline{u}) \\ + (\lambda^1 + \mu^1)\frac{\partial}{\partial t}\underline{\nabla}(\underline{\nabla} \cdot \underline{u}) + \mu\nabla^2\underline{u} + \mu^1\frac{\partial}{\partial t}\nabla^2\underline{u} - \gamma_0\underline{\nabla}\theta + \\ \mu_e\sigma(\underline{E} \times \underline{H_0}) + \sigma\mu_e^2\{(\underline{\dot{u}} \times \underline{\Omega} \times \underline{u}) \times \underline{H_0}\} \times \underline{H_0} - k_0\mu_e(\underline{\nabla}\theta \times \underline{H_0}), \end{aligned} \tag{11}$$

$$\nabla^2\underline{E} - \underline{\nabla}(\underline{\nabla} \cdot \underline{E}) = \mu_e\sigma[\underline{\dot{E}} + \mu_e(\underline{\ddot{u}} + \underline{\Omega} \times \underline{\dot{u}}) \times \underline{H_0}] - k_0\mu_e\underline{\nabla}\dot{\theta} + \mu_e \in \underline{\ddot{E}}. \tag{12}$$

In the absence of rotation and visco-elastic parameters, these equations are reduced to those reported in Ref. 8.

Let us introduce the following notations

$$\omega^* = \frac{\rho c_V c_1^2}{k},\ c_1^2 = \frac{\lambda+2\mu}{\rho},\ g = \frac{\gamma_0}{\rho c_V},\ b = \frac{\gamma_0\theta_0}{\mu},\ \beta^2 = \frac{\lambda+2\mu}{\mu},\ \beta^{1^2} = \frac{\lambda^1+2\mu^1}{\mu^1},$$
$$\tau = \tau_0\omega^*,$$

$$s = c_1^2 \in \mu_e,\ \in_\theta = \frac{bg}{\beta^2},\ \in_E = \frac{\mu_e\left|\underline{H_0}\right|^2}{\rho c_1^2},\ \nu = \frac{1}{\sigma\mu_e},\ \overline{\nu} = \frac{\nu\omega^*}{c_1^2},\ \overline{\nu^1} = \frac{\mu^1}{\mu}\omega^*\overline{\nu}$$

$$\Pi = \Pi_0\mu_e \in \omega^*\left|\underline{H_0}\right|/(g\rho c_V\theta_0),\ \overline{k} = gk_0\theta_0/\left|\underline{H_0}\right|.$$

Here $\underline{H_0} = \left|\underline{H_0}\right|\underline{n}$ where $\underline{n}$ is a unit vector in the direction of the primary uniform magnetic field. Using as in Ref. 8, $\frac{1}{\omega^*}, \frac{c_1}{\omega^*}, \frac{c_1}{g\omega^*}, \theta_0$, and $\left|\underline{H_0}\right|\mu_e c_1/g$ as the unit of time, length, displacement, temperature, and the electric field, respectively, we obtain the following non-dimensional field equations:

$$\dot{\theta} + \tau\ddot{\theta} - \nabla^2\theta + \dot{\Delta} + \tau\ddot{\Delta} = \Pi \cdot \underline{\nabla} \cdot \underline{\dot{E}}, \tag{13}$$

$$\overline{\nu}s\underline{\ddot{E}} + \underline{\dot{E}} + (\underline{\ddot{u}} + \underline{\Omega} \times \underline{\dot{u}}) \times \underline{n} - \overline{\nu}\overline{k}\underline{\nabla}\dot{\theta} = \overline{\nu}\nabla^2\underline{E} - \overline{\nu}\underline{\nabla}(\underline{\nabla} \cdot \underline{E}), \tag{14}$$

$$\begin{aligned} &\beta^2\overline{\nu}[\ddot{u}_i + (\underline{\Omega} \times (\underline{\Omega} \times \underline{u}))i + (2\underline{\Omega} \times \underline{\dot{u}})_i] \\ &= \overline{\nu}(\beta^2 - 1)\Delta_{,i} + \overline{\nu^1}(\beta'^2 - 1)\frac{\partial}{\partial t}\Delta_{,i} + \overline{\nu}\nabla^2 u_i + \overline{\nu^1}\frac{\partial}{\partial t}\nabla^2 u_i - \overline{\nu}\beta^2 \in_\theta \theta_{,i} \\ &+ \beta^2 \in_E \{(\underline{E} \times \underline{n}) + ((\underline{\dot{u}} + \underline{\Omega} \times \underline{u}) \times \underline{n}) \times \underline{n} - \overline{\nu}\overline{k}(\underline{\nabla}\theta \times \underline{n})\} \\ &\text{i} = 1,\ 2,\ 3\ldots\ldots. \end{aligned} \tag{15}$$

where "comma" followed by "i" in the subscripts of Δ and θ above indicate partial derivative with respect to the spatial coordinate. The field equations, Eqs. (13)–(15), without the visco-elastic effect ($\overline{\nu^1}=0,\ \beta^1=0$) are reduced to the corresponding equations of Roy Choudhuri [13, p. 521]. Here $\theta, \underline{u}, \underline{E}$ are all corresponding dimensionless quantities.

3. PLANE HARMONIC WAVES AND DISPERSION EQUATION

We consider, without any loss in generality, plane waves in the rotating medium in the x_1-direction. We look for time-varying dynamic solutions and as such, the time-independent part of the centripetal acceleration as well as body forces will be neglected.

We assume $\underline{u}=(u, \mathrm{v}, w), \underline{E}=(E_1, E_2, E_3), \underline{\Omega}=(\Omega_1, \Omega_2, \Omega_3)$.

The primary uniform magnetic field $\underline{H_0}$ is in the direction $\underline{H_0}=H_0\underline{n}$ where $\underline{n}=(n_1, o, n_3)$. Equations (13)–(15) are reduced to

$$\dot{\theta}+\tau\ddot{\theta}-\theta''+\dot{u}'+\tau\ddot{u}'=\pi\dot{E}_1', \tag{16}$$

$$\overline{\nu}s\ddot{E}_1+\dot{E}_1+n_3\ddot{\mathrm{v}}+n_3(\Omega_3\dot{u}-\Omega_1\dot{w})-\overline{\nu}\overline{k}\dot{\theta}'=0, \tag{17}$$

$$\overline{\nu}s\ddot{E}_2+\dot{E}_2+n_1\ddot{w}-n_3\ddot{u}+n_1(\Omega_1\dot{\mathrm{v}}-\Omega_2\dot{u})-n_3(\Omega_1\dot{w}-\Omega_3\dot{\mathrm{v}})=\overline{\nu}E_2'', \tag{18}$$

$$\overline{\nu}s\ddot{E}_3+\dot{E}_3+n_1\ddot{\mathrm{v}}-n_1(\Omega_3\dot{u}-\Omega_1\dot{w})=\overline{\nu}E_3'', \tag{19}$$

$$\begin{aligned}\overline{\nu}[\ddot{u}\ +\ &(\Omega_1u+\Omega_2\mathrm{v}+\Omega_3w)\Omega_1-\Omega^2u+2(\Omega_2\dot{w}-\Omega_3\dot{\mathrm{v}})]\\ &=\overline{\nu}u''+\overline{\nu'}\beta_1^2\dot{u}''-\overline{\nu}\in_\theta\theta'+\in_E\{E_2n_3+n_1n_3\dot{w}\\ &-n_3^2\dot{u}-\Omega_2n_3(n_1u+n_3w)+vn_3(n_1\Omega_1+n_3\Omega_3)\},\end{aligned} \tag{20}$$

$$\begin{aligned}&\overline{\nu}\beta^2[\ddot{\mathrm{v}}+(\Omega_1u+\Omega_2\mathrm{v}+\Omega_3w)\Omega_2-\Omega^2\mathrm{v}+2(\dot{u}\Omega_3-\dot{w}\Omega_1)]\\ &=\overline{\nu}\mathrm{v}''+\overline{\nu'}\dot{\mathrm{v}}''+\beta^2\in_E\{E_3n_1-E_1n_3-\dot{\mathrm{v}}-(n_1u+n_3\mathrm{v})(n_1\Omega_3-n_3\Omega_1)\\ &+(n_1\Omega_1+n_3\Omega_3)(n_1w-n_3u)+\overline{\nu}\overline{k}n_3\theta'\},\end{aligned} \tag{21}$$

$$\begin{aligned}&\overline{\nu}\beta^2[\ddot{w}+(\Omega_1u+\Omega_2\mathrm{v}+\Omega_3w)\Omega_3-\Omega^2w+2(\dot{\mathrm{v}}\Omega_1-\dot{u}\Omega_2)]\\ &=\overline{\nu}w''+\overline{\nu'}\dot{w}''-\beta^2\in_E n_1\times\{E_2+n_1\dot{w}-n_3\dot{u}-\Omega(n_1u+n_3w)\\ &+v(n_1\Omega_1+n_3\Omega_3),\end{aligned} \tag{22}$$

where $\beta_1^2 = \frac{\beta'^2}{\beta^2}$, $\overline{\nu_1} = \frac{\overline{\nu'}}{\overline{\nu}}$ and $\beta_1, \overline{\nu_1}$ are the visco-elastic parameters.

Here the prime over the field variables denotes the differentiation with respect to the x_1-coordinate.

Case A: We consider the case $n_3 = 0$, $n_1 = 1$. Equations (16)–(22) are reduced to

$$\dot{\theta} + \tau\ddot{\theta} - \theta'' + \dot{u}' + \tau\ddot{u}' = \Pi \dot{E}_1', \tag{23}$$

$$\overline{\nu} s \ddot{E}_1 + \dot{E}_1 - \overline{\nu}\overline{k}\dot{\theta}' = 0, \tag{24}$$

$$\overline{\nu} s \ddot{E}_2 + \dot{E}_2 + \ddot{w} + \Omega_1 \dot{\mathrm{v}} - \Omega_2 \dot{u} = \overline{\nu} E_2'', \tag{25}$$

$$\overline{\nu} s \ddot{E}_3 + \dot{E}_3 - \ddot{\mathrm{v}} - \Omega_3 \dot{u} + \Omega_1 \dot{w} = \overline{\nu} E_3'', \tag{26}$$

$$\begin{aligned} \ddot{u} + \Omega_1\Omega_2 \mathrm{v} - \Omega_2^2 u + \Omega_1\Omega_3 w - \Omega_3^2 u + 2(\Omega_2 \dot{w} - \dot{\mathrm{v}}\Omega_3) \\ = u'' + \overline{\nu_1}\beta_1^2 \dot{u}'' - \in_\theta \theta', \end{aligned} \tag{27}$$

$$\begin{aligned} \overline{\nu}\beta^2[\ddot{\mathrm{v}} + \Omega_3(w\Omega_2 - \mathrm{v}\Omega_3) + \Omega_1(\Omega_2 u - \Omega_1 \mathrm{v}) + 2(\dot{u}\Omega_3 - \dot{w}\Omega_1)] \\ = \overline{\nu} v'' + \overline{\nu'}\dot{\mathrm{v}}'' + \beta^2 \in_E \{E_3 - \dot{\mathrm{v}} - u\Omega_3 + \Omega_1 w\}, \end{aligned} \tag{28}$$

$$\begin{aligned} \overline{\nu}\beta^2[\ddot{w} + \Omega_1(u\Omega_3 - w\Omega_1) + \Omega_2(\Omega_3 \mathrm{v} - \Omega_2 w) + 2(\dot{\mathrm{v}}\Omega_1 - \dot{u}\Omega_2)] \\ = \overline{\nu} w'' + \overline{\nu'}\dot{w}'' - \beta^2 \in_E \{E_2 + w - \Omega_2 u + \mathrm{v}\Omega_1\}. \end{aligned} \tag{29}$$

When $\overline{\nu_1} = 0$, Eqs. (27)–(29) reduce to the corresponding equations of Roy Choudhuri [13, p. 522]. When $\Omega_1 = \Omega_2 = \Omega_3 = 0$, the system of equations, Eqs. (23)–(29), break up into three groups as

$$\dot{\theta} + \tau\ddot{\theta} - \theta'' + \dot{u}' + \tau\ddot{u}' = \Pi \dot{E}_1{}', \tag{30}$$

$$\overline{\nu} s \ddot{E}_1 + \dot{E}_1 - \overline{\nu}\overline{k}\dot{\theta}_1{}' = 0, \tag{31}$$

$$\ddot{u} = u'' + \overline{\nu_1}\beta_1^2 \dot{u}'' - \in_\theta \theta', \quad \text{and} \tag{32}$$

$$\overline{\nu} s \ddot{E}_2 + \dot{E}_2 + \ddot{w} = \overline{\nu} E_2{}'', \tag{33}$$

$$\overline{\nu} s \ddot{E}_3 + \dot{E}_3 - \ddot{\mathrm{v}} = \overline{\nu} E_3{}'', \tag{34}$$

$$\overline{\nu}\beta^2\ddot{\mathrm{v}} = \overline{\nu}v'' + \overline{\nu'}\dot{\mathrm{v}}'' + \beta^2 \in_E (E_3 - \dot{\mathrm{v}}), \tag{35}$$

$$\overline{\nu}\beta^2\ddot{w} = \overline{\nu}w'' + \overline{\nu'}\dot{w}'' - \beta^2 \in_E (E_2 + w). \tag{36}$$

Equations (30)–(32) correspond to coupled thermo-visco-elastic dilatational–electrical waves. Equation (33) with (36) and Eq. (34) with (35) correspond to coupled visco-elastic shear electrical waves, not studied earlier. These three groups are independent of each other. The coupling between the longitudinal and shear motions disappears when $\Omega_1 = \Omega_2 = \Omega_3 = 0$. When $\Omega_2 = \Omega_3 = 0, \Omega_1 = \Omega \neq 0$, Eqs. (23)–(29) are reduced to

$$\dot{\theta} + \tau\ddot{\theta} - \theta'' + \dot{u}' + \tau\ddot{u}' = \Pi\dot{E_1}', \tag{37}$$

$$\overline{\nu}s\ddot{E}_1 + \dot{E}_1 - \overline{\nu}\overline{k}\dot{\theta}_1{}' = 0, \tag{38}$$

$$\ddot{u} = u'' + \overline{\nu_1}\beta_1^2\dot{u}'' - \in_\theta \theta', \tag{39}$$

$$\overline{\nu}s\ddot{E}_2 + \dot{E}_2 + \ddot{w} + \Omega\dot{\mathrm{v}} = \overline{\nu}E_2{}'', \tag{40}$$

$$\overline{\nu}s\ddot{E}_3 + \dot{E}_3 - \ddot{\mathrm{v}} + \dot{w}\Omega = \overline{\nu}E_3{}'', \tag{41}$$

$$\overline{\nu}\beta^2[\ddot{\mathrm{v}} - \Omega^2 v - 2\dot{w}\Omega] = \overline{\nu}\mathrm{v}'' + \overline{\nu}'\dot{\mathrm{v}}'' + \beta^2 \in_E \{E_3 - \dot{\mathrm{v}} - \Omega_w\}, \tag{42}$$

$$\overline{\nu}\beta^2[\ddot{w} - w\Omega^2 + 2\Omega\dot{\mathrm{v}}] = \overline{\nu}w'' + \overline{\nu}'\dot{w}'' - \beta^2 \in_E \{E_2 + \dot{w} + \mathrm{v}\Omega\}. \tag{43}$$

Equations (37)–(39) form one system I, which corresponds to the coupled visco-elastic-thermal-dilatational and electrical waves. The four remaining equations, Eqs. (40)–(43), form a second system II, which corresponds to coupled electrical visco-elastic shear waves (independent of the thermal field as expected) and x_2, x_3 components of the electric field, influenced by rotation.

3.1. Dispersion Equations for the Systems II and I

For waves propagation in the x_1-direction, we write

$$(E_2,\ E_3,\ v,\ w) = a_{E_2},\ a_{E_3},\ a_V,\ a_\omega)\ \exp\{i(qx_1 + \omega t)\}, \tag{44}$$

where ω is real and q is complex. The wave speed is then $C = \omega/\mathrm{Re}(q)$ and the attenuation coefficient $S = -\mathrm{Im}\ (q)$.

We substitute Eq. (44) into Eqs. (40)–(43) and for non-trivial solutions for $a_{E_2}, a_{E_3}, a_V, a_\omega$ we obtain the following dispersion relation for the system II:

$$\begin{vmatrix} i\omega-\overline{\nu}s\omega^2+\overline{\nu}q^2, & 0, & \Omega i\omega, & -\omega^2 \\ 0, & i\omega-\overline{\nu}s\omega^2+\overline{\nu}q^2, & \omega^2, & i\omega\Omega \\ 0, & -\beta^2\in_E, & \begin{matrix}\overline{\nu}\beta^2(-\omega^2-\Omega^2)+\overline{\nu}q^2\\ +i\omega q^2\overline{\nu'}+\beta^2\in_E i\omega,\end{matrix} & -2\overline{\nu}\beta^2 i\omega\Omega-\beta^2\in_E\Omega \\ \beta^2\in_E, & 0, & 2\overline{\nu}\beta^2\Omega i\omega+\beta^2\in_E\Omega, & \begin{matrix}\overline{\nu}\beta^2(-\omega^2-\Omega^2)\\ +\overline{\nu}q^2+i\omega q^2\overline{\nu'}+\beta^2\in_E i\omega\end{matrix} \end{vmatrix}=0 \tag{45}$$

Equation (45) represents a more general dispersion relation in the sense that it incorporates the effects of visco-elasticity and the rotation on the phase speed of the waves when the displacement current and the charge density are included in the analysis.

For system I, we substitute $(\theta, E_1, u) = (a_\theta, a_{E_1}, a_u)\exp\{i(qx_1+\omega t)\}$ into Eqs. (37)–(39) and for non-trivial solutions for constants a_θ, a_{E_1}, a_u we obtain dispersion equation for the system I as

$$\begin{vmatrix} i\omega-\tau\omega^2+q^2, & \Pi q\omega, & -\omega q-\tau\omega^2 iq \\ \overline{\nu}\overline{k}q\omega, & i\omega-\overline{\nu}s\omega^2, & 0 \\ \in_\theta iq, & 0, & q^2-\omega^2+\overline{\gamma_1}\beta_1^2 i\omega q^2 \end{vmatrix} = 0. \tag{46}$$

4. PERTURBATION SOLUTION OF THE SYSTEM II AND I FOR SMALL $\in_E$ AND $\in_\theta$

Expanding the determinant of Eq. (45), we obtain for the system II,

$$\begin{aligned}
&(i\omega-\overline{\nu}s\omega^2+\overline{\nu}q^2)[i\omega-\overline{\nu}s\omega^2+\overline{\nu}q^2)\{(\overline{\nu}\beta^2(-\omega^2-\Omega^2)\\
&\quad+\overline{\nu}q^2+i\omega q^2\overline{\nu'}+\beta^2\in_E i\omega)^2+(2\overline{\nu}\beta^2 i\omega\Omega+\beta^2\in_E\Omega)^2\}\\
&\quad+\beta^2\in_E\{\omega^2(\overline{\nu}\beta^2(-\omega^2-\Omega^2)+\overline{\nu}q^2\\
&\quad++i\omega q^2\overline{\nu'}+\beta^2\in_E i\omega)-i\omega\Omega(2\overline{\nu}\beta^2\Omega i\omega+\beta^2\in_E\Omega)\}]\\
&\quad-\beta^2\in_E[-\beta^2\in_E(-\Omega^2\omega^2+\omega^4)-(i\omega-\overline{\nu}s\omega^2+\overline{\nu}q^2)\\
&\quad\{\Omega i\omega(-2\overline{\nu}\beta^2 i\omega\Omega-\beta^2\in_E\Omega)+\omega^2(\overline{\nu}\beta^2(-\omega^2-\Omega^2)+\overline{\nu}q^2+i\omega q^2\overline{\nu'}\\
&\quad+\beta^2\in_E i\omega w)\}]=0.
\end{aligned} \tag{47}$$

To obtain the perturbation solution for II, for small $\in_E$ we first put $\in_E = 0$ in Eq. (47).

Then we obtain $(q^2 - s^*\omega^2)$ $\left[\left\{q^2(1 + i\omega\overline{\nu_1}) - \beta^2(\omega^2 + \Omega^2)\right\}^2 - 4\beta^4\Omega^2\omega^2\right] = 0$, where we write $i\omega - \overline{\nu}s\omega^2 + \overline{\nu}q^2 = \overline{\nu}(q^2 - s^*\omega^2)$, $\quad s^* = s - \frac{i}{\overline{\nu}\omega}$.

The above equation has two solutions: $q^2 = s^*\omega^2 = q_{E_0}^2$, $q^2 = \beta^{*2}\omega_0^2 = q_{\nu_0}^2$, where $\omega_0 = \omega + \Omega$ and $\beta^{*2} = \frac{\beta^2}{1 + i\omega\overline{\nu_1}}$.

Next let us write Eq. (47) in a form neglecting $0\,(\in_E^2)$ and substitute into this new equation for q^2, the following:

$$q_E^2 = q_{E_0}^2 + \xi_E \in_E + 0(\in_E^2) = s^*\omega^2 + \xi_E \in_E + 0(\in_E^2),$$

$$q_\nu^2 = q_{\nu_0}^2 + \xi_\nu \in_E + 0(\in_E^2) = \beta^*\omega_0^2 + \xi_\nu \in_E + 0(\in_E^2),$$

where ξ_E, ξ_ν are to be determined. Equating like powers of $\in_E$, after lengthy algebra we obtain,

$$\xi_E = \frac{2\beta^2\omega^2}{\overline{\nu}^2} \cdot \frac{\left\{\beta^2\left(\omega^2 + \Omega^2\right) - s^*\omega^2\left(1 + i\omega\overline{\nu_1}\right) - 2\beta^2\Omega^2\right\}}{\left[\left\{\beta^2\left(\omega^2 + \Omega^2\right) - s^*\omega^2\left(1 + i\omega\overline{\nu_1}\right)\right\}^2 - 4\beta^4\Omega^2\omega^2\right]}.$$

For $\overline{\nu_1} = 0$ this agrees with the corresponding result derived by Roy Choudhuri [13] except some erroneous calculation in the last term. Furthermore, for $\overline{\nu_1} = 0$ and $\Omega = 0$, the result agrees with the corresponding result, in the non-rotating case as reported in Ref. 8 [Eq. (31b)]. Similarly substituting $q^2 = q_\nu^2$, we obtain

$$\xi_\nu = -\frac{\beta^2(\omega + \Omega)(i\overline{\nu}\psi_1 + \omega)}{\overline{\nu}^2(1 + i\omega\overline{\nu_1})\psi_1} \quad \text{where } \psi_1 = \beta^{*2}\omega_0^2 - s^*\omega^2.$$

For $\overline{\nu_1} = 0$ and $\psi_1 = \beta^2\omega_0^2 - s^*\omega^2$, the above result agrees with the corresponding result of Roy Choudhuri [13]. For $\overline{\nu_1} = 0, \Omega = 0$ we have

$$\psi_1 = \beta^2\omega^2 - s^*\omega^2 \quad \text{and} \quad \xi_\nu = \frac{\beta^2\{i + \overline{\nu}\omega(s^* - \beta^2)\}}{i\overline{\nu}^2(s^* - \beta^2)}.$$

This agrees with the corresponding result, in the non-rotating case as reported in Ref. 8 [Eq. (31a)].

Thus, we obtain the following perturbation solution for the system II as

$$q_E^2 = s^*\omega^2\left[1 + \frac{2\beta^2 \in_E}{s^*\overline{\nu}^2}\frac{\{\beta^2(\omega^2 - \Omega^2) - s^*\omega^2(1 + i\omega\overline{\nu_1})\}}{\{(\beta^2(\omega^2 + \Omega^2) - s^*\omega^2(1 + i\omega\overline{\nu_1}))^2 - 4\beta^4\Omega^2\omega^2\}}\right]$$

$$q_v^2 = \beta^{*2}\omega_0^2\left[1-\{(\omega+\Omega)(i\overline{\nu}\psi_1+\omega)\}\in_E/\psi_1\overline{\nu}^2\omega_0^2\right]. \tag{48}$$

4.1. Perturbation Solution for the System I

The dispersion relation, Eq. (46), can be written in the form,

$$\{\omega^2 - q^2(1+\overline{\nu_1}\beta_1^2 i\omega)\}[\overline{k}\Pi q^2 + s^*(q^2-\omega^2\tau^*)] + \in_\theta q^2\omega^2\tau^* s^* = 0,$$

where $\tau^* = \tau - \frac{i}{\omega}$. For $\overline{\nu_1} = 0$, this equation readily agrees with that reported in Ref. 13.

In this case the perturbation solution for small $\in_\theta$ is given by

$$q_u^2 = \omega^2/(1+\overline{\nu_1}\beta_1^2 i\omega) + \xi_u \in_\theta + 0(\in_\theta^2),$$

$$q_\theta^2 = \frac{\tau^*\omega^2 s^*}{s^* + \overline{k}\Pi} + \xi_\theta . \in_\theta + 0(\in_\theta^2), \tag{49}$$

where

$$\xi_u = \frac{\omega^2\tau^* s^*}{[(1-\tau_1)s^* + \overline{k}\Pi](1+\overline{\nu_1}\beta_1^2 i\omega)}, \quad \xi_\theta = \frac{\tau^{*2}\omega^2 s^{*2}}{\{(\tau_1-1)s^* - \overline{k}\Pi\}(s^* + \overline{k}\Pi)}$$

where $\tau_1 = \tau^*(1+\overline{\nu_1}\beta_1^2 i\omega)$. For $\overline{\nu_1} = 0, \tau_1 = \tau^*$ the above result agrees with the corresponding result as reported in Ref. 13.

Case B: Charge density vanishes. In this case $\rho_e = 0, \in \neq 0, \overline{\nu}\overline{k} \neq 0$

Then Eqs. (16)–(22), in the case $\Omega_2 = \Omega_3 = 0, \Omega_1 = \Omega \neq 0$ are reduced to

$$\begin{aligned}
&\overline{\nu}\ddot{u} = \overline{\nu}u'' + +\overline{\nu'}\beta_1^2\dot{u}'' - \overline{\nu}\in_\theta\theta' + \in_E n_3(E_2 + n_1\dot{w} - n_3\dot{u} + \mathrm{v}\Omega n_1)\\
&\overline{\nu}\beta^2[\ddot{\mathrm{v}} - \Omega^2\mathrm{v} - 2\dot{w}\Omega] = \overline{\nu}v'' + \overline{\nu'}\dot{\mathrm{v}}'' + \beta^2\in_E(E_3 n_1 - \dot{\mathrm{v}}n_1^2 + w\Omega n_1^2)\\
&\overline{\nu}\beta^2[\ddot{w} - \Omega^2 w + 2\Omega\dot{\mathrm{v}}] = \overline{\nu}w'' + \overline{\nu'}\dot{w}'' - \beta^2\in_E n_1(E_2 + n_1\dot{w} - n_3\dot{u} + \mathrm{v}n_1\Omega)\\
&\dot{\theta} + \tau\ddot{\theta} - \theta'' + \dot{u}' + \tau\ddot{u}' = 0\\
&\overline{\nu}s\ddot{E}_2 + \dot{E}_2 + n_1\ddot{w} - n_3\ddot{u} + n_1\Omega\dot{\mathrm{v}} = \overline{\nu}E_2''\\
&\overline{\nu}s\ddot{E}_3 + \dot{E}_3 - n_1\ddot{\mathrm{v}} + n_1\Omega\dot{w} = \overline{\nu}E_3''
\end{aligned} \tag{50}$$

If $n_1 = 0$, $n_3 = 1$, the last equation uncouples and the other five equations of Eq. (50) reduce to two independent systems I_D and II_D. The system I_D consists of the equations,

$$\begin{aligned}&\dot{\theta} + \tau\ddot{\theta} - \theta'' + u' + \tau\ddot{u}' = 0\\&\overline{\nu}\ddot{u} = \overline{\nu}u'' + \overline{\nu'}\beta_1^2\dot{u}'' - \overline{\nu}\in_\theta \theta' + \in_E (E_2 - \dot{u})\\&\overline{\nu}s\ddot{E}_2 + \dot{E}_2 - \ddot{u} = \overline{\nu}E_2''.\end{aligned} \tag{51}$$

The system II_D consists of the equations,

$$\beta^2[\ddot{\mathrm{v}} - \Omega^2\mathrm{v} - 2\dot{w}\Omega] = \mathrm{v}'' + \overline{\nu_1}\dot{\mathrm{v}}'', \beta^2[\ddot{w} - \Omega^2 w + 2\Omega\dot{\mathrm{v}}] = w'' + \overline{\nu_1}\dot{w}''. \tag{52}$$

These equations correspond to the visco-elastic shear waves modified by rotation of the medium. The solution of the dispersion equation is $q = \frac{\beta(\omega+\Omega)}{(1+\overline{\nu}_1^2\omega^2)^{1/2}}(1 - \overline{\nu}i\omega)^{1/2}$.

The phase velocity $c = (\omega/Re(q)) = \frac{\sqrt{2}\omega(1+\overline{\nu}_1^2\omega^2)^{1/2}}{\beta(\omega+\Omega)[\sqrt{1+\overline{\nu}_1^2\omega^2}+1]^{1/2}}$.

For $\overline{\nu_1} = 0$, this readily agrees with that derived in Ref. 13 (p. 526).

In the system I_D, we put $(\theta, u, E_2) = (a_\theta, a_u, a_{E_2})\exp\{i(qx + \omega t)\}$.

For non-zero solution for the constants a_θ, a_u, a_{E_2} we obtain the characteristic equation,

$$\begin{vmatrix} -\tau\omega^2 + q^2 + i\omega, & -q\omega - \tau i\omega^2 q, & 0 \\ \overline{\nu}\in_\theta iq, & -\overline{\nu}\omega^2 + \overline{\nu}q^2 + \overline{\nu^1}\beta_1^2 q^2\omega i + \in_E i\omega, & -\in_E \\ 0, & \omega^2, & \overline{\nu}q^2 - \overline{\nu}s\omega^2 + i\omega \end{vmatrix} = 0,$$

which can be written in the form,

$$\begin{aligned}(q^2 - \omega^2\tau^*)[\{\overline{\nu}(q^2 - \omega^2 + \overline{\nu_1}\beta_1^2 i\omega) + \in_E i\omega\}\overline{\nu}(q^2 - \omega^2 s^*) + \in_E \omega^2]\\ - \overline{\nu}^2 \in_\theta q^2\omega^2\tau^*(q^2 - \omega^2 s^*) = 0.\end{aligned} \tag{53}$$

4.2. Perturbation Solution for Small $\in_E$ and $\in_\theta$

We first put $\in_E = 0$ in Eq. (53), which then becomes

$$(q^2 - \omega^2 s^*)[(q^2 - \omega^2\tau^*)(q^2 - \omega^2 + \overline{\nu_1}\beta_1^2 i\omega) - \in_\theta q^2\omega^2\tau^*] = 0.$$

The solution, $q^2 = s^*\omega^2 = q_{E_2}^2$, corresponds to the uncoupled electrical wave. The other solutions, corresponding to the second factor are found by perturbation method for small $\in_\theta$ to be

$$q_1^2 = \omega^2\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\left[1 + \frac{\tau^*}{(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2) - \tau^*} \in_\theta\right],$$

$$q_2^2 = \omega^2\tau^*\left[1 - \frac{\tau^*}{(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2) - \tau^*} \in_\theta\right].$$

For $\overline{\nu}_1 = 0$ (i.e., without visco-elastic effect), these agrees with Ref. 13 (p. 526).

These correspond to the coupled visco-elastic dilatational thermal waves. For the coupled waves, we next put

$$q^2 = q_u^2 = q_1^2 + \xi_u \in_E + 0(\in_E^2),\ \ q^2 = q_\theta^2 = q_2^2 + \xi_\theta \in_E + 0(\in_E^2),$$
$$q^2 = s^*\omega^2 + \xi_E \in_E + 0(\in_E^2),$$

in Eq. (53) and equate like powers of $\in_E$ to obtain after lengthy algebra,

$$q_u^2 = \omega^2\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\left[1 + \frac{\tau^*}{\{(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2) - \tau^*\}}\right]$$
$$- \frac{\in_E}{\{(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2) - \tau^*\}^2}\left[\left\{\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right\}^2 + \left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) \in_\theta \tau^*\right]$$
$$\left[\left\{\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right\}^2 + i\omega\overline{\nu}\left\{\left(\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - s^*\right)\left(\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right)\right.\right.$$
$$\left.\left. + \left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) \in_\theta \tau^*\right\}\right] / M_u,$$

where

$$M_u = \overline{\nu^2}\left[\frac{\tau^*\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) \in_\theta}{\left\{\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right\}^2}\left\{\left(\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right)^2 + \left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) \in_\theta \tau^*\right\}\right.$$
$$+ \frac{1}{\left\{\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right\}^2}\left\{\left(\left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) - \tau^*\right)^2 + \left(1 - \frac{i}{\omega}\overline{\nu_1}\beta_1^2\right) \in_\theta \tau^*\right\}$$

$$\left\{\left(\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-s^*\right)\left(\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right)+\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\in_\theta\tau^*\right\}$$

$$+\frac{\tau^*\in_\theta}{\left\{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right\}^2}\left\{\left(\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-s^*\right)\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*)+\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\in_\theta\tau^*\right\}$$

$$\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\in_\theta\left\{\left(2\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-s^*\right)+\frac{2\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\in_\theta\tau^*}{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*}\right\}\Bigg]$$

$$q_\theta^2=\omega^2\tau^*\left\{1-\frac{\tau^*\in_\theta}{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*}\right\}+\frac{\in_E\tau^{*2}\in_\theta}{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*}$$
$$\times\left[1+\frac{i\omega\overline{\nu}}{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*}\left\{(\tau^*-s^*)\left(\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right)-\tau^{*2}\in_\theta\right\}\right]\Big/M_\theta$$

and

$$M_\theta=\overline{\nu}^2\left[\frac{\tau^{*2}\in_\theta}{\{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*\}^2}\left\{\left\{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right\}^2+\tau{*}^2\in_\theta\right\}\right.$$
$$-\frac{\tau^{*2}\in_\theta}{\{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*\}^2}\left\{(\tau^*-s^*)\left\{\left\{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^{*2}\in_\theta\right\}\right\}\right.$$
$$-\frac{1}{\{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*\}^2}\left\{\left\{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right\}^2+\tau^{*2}\in_\theta\right\}$$
$$\times\left\{(\tau^*-s^*)\left\{\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)-\tau^*\right\}-\tau^{*2}\in_\theta\right\}-\frac{\in_\theta\tau^*}{\{(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*\}}$$
$$\left.\times\left\{(2\tau^*-s^*)\left((1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2)-\tau^*\right)-2\tau^{*2}\in_\theta\right\}\right]$$

$$q_{E_2}^2=s^*\omega^2+\frac{\in_E(\tau^*-s^*)}{\overline{\nu}^2\left[(s^*-\tau^*)\left\{s^*-\left(1-\frac{i}{\omega}\overline{\nu_1}\beta_1^2\right)\right\}-\in_\theta\tau^*s^*\right]}\tag{54}$$

to the first order in $\in_E$.

APPENDIX

A close look at the solutions given by Eqs. (48), (49), and (54) reveals that, in general, there are five different modified waves which can propagate in an unbounded isotropic electrically and thermally conducting visco-elastic medium. They are visco-elastic dilatational, shear, electro-magnetic, and thermal waves, i.e., visco-elastic dilatational, shear modes, thermal mode, and electrical mode (two components).

For small $\in_E$, the solutions of Eq. (33) are written as

$$q_v \approx \beta^* \omega_0 \left[1 - \frac{1}{2}\frac{\in_E}{\overline{\nu}^2 \omega_0^2}\frac{L}{\psi_1}\right], \quad q_E \approx \sqrt{s^*}\omega\left[1 + \frac{\beta^2 \in_E}{\overline{\nu}^2} \cdot \frac{M - 2\beta^2 \Omega^2}{s^*(M^2 - 4\beta^4 \Omega^2 \omega^2)}\right]$$

where

$$L = (\omega + \Omega)(\omega + i\overline{\nu}\psi_1),\, M = \beta^2(\omega^2 + \Omega^2) - s^*\omega^2(1 + i\omega\overline{\nu}_1)$$

$$R_e(q_v) \approx \frac{\beta\omega_0}{\sqrt{2}(1 + \omega^2\overline{\nu}_1^2)^{1/2}}\left[L_1 - \frac{\in_E}{2\overline{\nu}\omega_0}(L_1 L_3 - L_2 L_4)\right]$$

$$\mathrm{Im}(q_v) \approx \frac{\beta\omega_0}{\sqrt{2}(1 + \omega^2\overline{\nu}_1^2)^{1/2}}\left[L_2 - \frac{\in_E}{2\overline{\nu}\omega_0}(L_2 L_3 + L_1 L_4)\right],$$

where $L_{1,2} = \{(1 + \omega^2\overline{\nu}_1^2)^{1/2} \pm 1\}^{1/2}$, $L_3 = \frac{s\omega^3\overline{\nu}^1(\overline{\nu}p - \omega^2\overline{\nu}_1) + \overline{\nu}\omega p q}{(\overline{\nu}p - \omega^2\overline{\nu}_1)^2 + q^2\omega^2}$

$$L_4 = \frac{\overline{\nu}p(\overline{\nu}p - \omega^2\overline{\nu}_1) - qs\omega^4\overline{\nu}^1}{(\overline{\nu}p - \omega^2\overline{\nu}_1)^2 + q^2\omega^2}, \quad p = \beta^2\omega_0^2 - s\omega^2, q = (1 - s\omega^2\overline{\nu}^1).$$

The phase velocity $c_v = \frac{\omega}{Re(q_v)} \approx \frac{\omega\sqrt{2}(1+\omega^2\overline{\nu}_1^2)^{1/2}}{\beta\omega_0 L_1}\left[1 + \frac{\in_E}{2\overline{\nu}\omega_0 L_1}(L_1 L_3 - L_2 L_4\right]$.

The attenuation co-efficient $s_v \approx -\frac{\beta\omega_0}{\sqrt{2}(1+\omega^2\overline{\nu}_1^2)^{1/2}}\left[L_2 - \frac{\in_E}{2\overline{\nu}\omega_0}(L_2 L_3 + L_1 L_4)\right]$.

In the absence of the visco-elastic effect, $\overline{\nu}^1 = \overline{\nu}_1 = 0$

$$q = 1,\ L_1 = \sqrt{2},\ L_2 = 0,\ L_3 = \frac{\overline{\nu}\omega p}{\overline{\nu}^2 p^2 - \omega^2},\ L_4 = \frac{\overline{\nu}^2 p^2}{\overline{\nu}^2 p^2 + \omega^2},$$

and the above results are identical with the similar results of Roy Choudhuri [13, p. 528].

For large ω,

$$L_{1,2} \to \sqrt{\omega\overline{\nu}_1},\ L_3 \to 0,\ L_4 \to 1, \qquad Re(q_v) \approx \frac{\beta\sqrt{\omega}}{\sqrt{2\overline{\nu}_1}}, \quad \mathrm{Im}(q_v) \approx \frac{\beta\sqrt{\omega}}{\sqrt{2\overline{\nu}_1}}$$

Hence $c_v \approx \frac{\sqrt{2\omega\overline{\nu}_1}}{\beta}$, $S_v \approx -\frac{\beta\sqrt{\omega}}{\sqrt{2\overline{\nu}_1}}$ for large ω.

For large ω the specific energy loss is given by $\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega}c_v s_v\right| \approx 4\pi$.

For small ω, $L_1 = \sqrt{2}$, $L_2 = 0$, $L_3 = 0$, $L_4 = 1$, $p = \beta^2\Omega^2$, $q = 1$.

$Re(q_v) \approx \beta\Omega$, $\mathrm{Im}(q_v) \approx -\frac{\beta}{2\overline{\nu}} \in_E$, Hence $c_v \approx \frac{\omega}{\beta\Omega}$, $s_v \approx \frac{\beta\in_E}{2\overline{\nu}}$ for small ω.

The specific energy loss is $\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_v s_v\right| \approx \frac{2\pi\in_E}{\Omega\overline{\nu}}$ for small ω.

$$R_e(q_E) \approx \sqrt{\frac{\omega}{2\overline{\nu}}}\left[M_1 + \frac{\beta^2\in_E}{\overline{\nu}^2}(M_1N_1 - M_2N_2)\right],$$

$$\mathrm{Im}(q_E) \approx \sqrt{\frac{\omega}{2\overline{\nu}}}\left[M_2 + \frac{\beta^2\in_E}{\overline{\nu}^2}(M_2N_1 + M_1N_2)\right],$$

where $M_{1,2} = \left[\sqrt{1 + s^2\overline{\nu}^2\omega^2} \pm s\overline{\nu}\omega\right]^{1/2}$.

$$N_1 = \frac{\left[\omega^2 M^*\left\{sM_0^* + \frac{2M_0}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})\right\} + \frac{\omega^2}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})\{2M_0\omega^2 s(1 - s\omega^2\overline{\nu^1}) - M_0^*\}\right]}{\left[\omega^2\left\{sM_0^* + \frac{2M_0}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})\right\}^2 + \frac{1}{\overline{\nu}^2}\{2M_0\omega^2 s(1 - s\omega^2\overline{\nu^1}) - M_0^*\}^2\right]}$$

$$N_2 = \frac{\left[\frac{\omega^3}{\overline{\nu}}(1 - s\omega^2\overline{\nu}^1)\left\{sM_0^* + \frac{2M_0}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})\right\} - M^*\left\{\frac{2M_0\omega^3 s}{\overline{\nu}}(1 - s\omega^2\overline{\nu^1}) - \frac{M_0^*\omega}{\overline{\nu}}\right\}\right]}{\left[\omega^2\left\{sM_0^* + \frac{2M_0}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})\right\}^2 + \frac{1}{\overline{\nu}^2}\left\{2M_0\omega^2 s(1 - s\omega^2\overline{\nu^1}) - M_0^*\right\}^2\right]}$$

$$M_0 = \beta^2(\omega^2 + \Omega^2) - \omega^2\left(s^2 + \frac{\overline{\nu_1}}{\overline{\nu}}\right)$$

$$M_0^* = M_0^2 - \frac{\omega^2}{\overline{\nu}^2}(1 - s\omega^2\overline{\nu^1})^2 - 4\beta^4\Omega^2\omega^2, \quad M^* = \beta^2(\omega^2 - \Omega^2) - \omega^2\left(s + \frac{\overline{\nu_1}}{\overline{\nu}}\right)$$

For small ω,

$$M_0 \sim \beta^2\Omega^2,\ M_0^* \sim \beta^4\Omega^4,\ M^* \sim -\beta^2\Omega^2, \quad M_{1,2} \sim 1,\ N_1 \sim 0,\ N_2 \sim -\frac{\omega\overline{\nu}}{\beta^2\Omega^2}$$

$$Re(q_E) \approx \sqrt{\frac{\omega}{2\overline{\nu}}}\left[1 + \frac{\in_E\omega}{\overline{\nu}\Omega^2}\right], \quad \mathrm{Im}(q_E) \approx \sqrt{\frac{\omega}{2\overline{\nu}}}\left[1 - \frac{\in_E\omega}{\overline{\nu}\Omega^2}\right]$$

Hence, $c_E \approx \sqrt{2\overline{\nu}\omega}\left[1 - \frac{\in_E\omega}{\overline{\nu}\Omega^2}\right]$, $s_E \approx -\sqrt{\frac{\omega}{2\overline{\nu}}}\left[1 - \frac{\in_E\omega}{\overline{\nu}\Omega^2}\right]$

$$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega}c_E s_E\right| \approx 4\pi\left[1 - \frac{\in_E\omega}{\overline{\nu}\Omega^2}\right]^2$$

Therefore, the specific energy loss depends on rotation but is independent of visco-elastic parameters at a low frequency.

For large ω, $M_1 \to \sqrt{2\omega s\overline{\nu}}$, $M_2 \to \frac{1}{\sqrt{2s\omega\overline{\nu}}}$, $N_{1,2} \to 0$.

$$Re(q_E) \approx \omega\sqrt{s}, \quad \mathrm{Im}(q_E) \approx \frac{1}{2\overline{\nu}\sqrt{s}}, \quad c_E \approx \frac{1}{\sqrt{s}} \text{ and } s_E \approx -\frac{1}{2\overline{\nu}\sqrt{s}} \text{ for large } \omega.$$

$$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_E s_E\right| \quad \approx \frac{2\pi}{\omega\overline{\nu}s}.$$

For small $\in_\theta$, Eq. (49) leads to $q_u \approx \frac{\omega(1-\overline{\nu_1}\beta_1^2 i\omega)^{1/2}}{(1+\overline{\nu_1^2}\beta_1^4\omega^2)^{1/2}}\left[1+\frac{1}{2}\frac{\tau^* s^*}{(1-\tau_1)s^*+\overline{k}\Pi}\in_\theta\right]$.

$$Re(q_u) \approx \frac{\omega}{\sqrt{2}(1+\overline{\nu_1^2}\beta_1^4\omega^2)^{1/2}}\left[M_3 + \frac{\in_\theta}{2}(M_3N_3 - M_4N_4)\right]$$

$$\mathrm{Im}(q_u) \approx \frac{\omega}{\sqrt{2}(1+\overline{\nu_1^2}\beta_1^4\omega^2)^{1/2}}\left[M_4 + \frac{\in_\theta}{2}(M_4N_3 - M_3N_4)\right],$$

where $M_{3,4} = (\sqrt{1+\overline{\nu_1^2}\beta_1^4\omega^2} \pm 1)^{1/2}$

$$N_3 = \frac{N_y D_y + N_m D_m}{D_y^2 + D_m^2}, \quad N_4 = \frac{N_m D_y - N_y D_m}{D_y^2 + D_m^2}, \quad N_y = \tau s\overline{\nu}\omega^2 - 1, \quad N_m = -\omega(s\overline{\nu} + \tau)$$

$$D_y = s\overline{\nu}\omega^2(1-\tau) + 1 + \overline{\nu}\omega^2\overline{k}\Pi - (s\overline{\nu}+\tau)\overline{\nu_1}\beta_1^2\omega^2, \quad D_m = \omega\{s\overline{\nu} - 1 + \tau - (s\overline{\nu}\tau\omega^2 + 1)\overline{\nu_1}\beta_1^2\}.$$

For large ω, $M_{3,4} \approx \sqrt{\omega\overline{\nu_1}}\beta_1$, N_3, $N_4 \approx 0$, $Re(q_u) \approx \frac{\sqrt{\omega}}{\sqrt{2\overline{\nu_1}}\beta_1}$, $Im(q_u) = \frac{\sqrt{\omega}}{\sqrt{2\overline{\nu_1}}\beta_1}$

Hence, $c_u \approx \sqrt{2\omega\overline{\nu_1}}\beta_1$ and $s_u = -\frac{\sqrt{\omega}}{\sqrt{2\overline{\nu_1}}\beta_1}$

$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_u s_u\right| \approx 4\pi$ for large ω.

For small ω,

$$M_3 \to \sqrt{2}, \quad M_4 \to \frac{1}{\sqrt{2}}\overline{\nu_1}\beta_1^2\omega, \quad D_y \to 1, D_m \to \omega(s\overline{\nu} - 1 + \tau - \overline{\nu_1}\beta_1^2)$$

$$N_y \to -1, \quad N_m \to -\omega(s\overline{\nu} + \tau), \quad N_3 \to -1, \quad N_4 \to -\omega(1 + \overline{\nu_1}\beta_1^2)$$

$$Re(q_u) \approx \omega\left(1 - \frac{\in_\theta}{2}\right), \quad \mathrm{Im}(q_u) \approx \frac{\omega^2}{4}\left[2\overline{\nu_1}\beta_1^2 - \in_\theta (2 + 3\overline{\nu_1}\beta_1^2)\right]$$

$$c_u \approx 1 + \frac{1}{2} \in_\theta, \quad s_u \approx -\frac{\omega^2}{4}\left[2\overline{v_1}\beta_1^2 - \in_\theta (2 + 3\overline{v_1}\beta_1^2\right]$$

indicating that at low frequencies, modified visco-elastic-thermo dilatational waves are affected by both electro-magnetic, visco-elastic parameters and thermo-elastic coupling.

$$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_u s_u\right| \approx \Pi\omega\left(1 + \frac{1}{2}\in_\theta\right)\left[2\overline{v_1}\beta_1^2 - \in_\theta\left(2 + 3\overline{v_1}\beta_1^2\right)\right]$$

Again, for small $\in_\theta$ from Eq. (49),

$$q_\theta = \frac{\sqrt{\omega}}{\sqrt{2}\left[\overline{v^2}\omega^2\left(s + \bar{k}\Pi\right)^2 + 1\right]^{1/2}} \left[\left\{M_5 - \frac{1}{2}\in_\theta (M_5N_3 - M_6N_4)\right\} + \left\{M_6 - \frac{1}{2}\in_\theta (M_6N_3 + M_5N_4)\right\} i\right]$$

$$q_\theta = \frac{\omega(\tau^* s^*)^{1/2}}{(s^* + \bar{k}\Pi)^{1/2}}\left[1 - \frac{1}{2}\frac{\tau^* s^*}{(1 - \tau_1)s^* + \bar{k}\Pi}\in_\theta\right]$$

$$Re(q_\theta) = \frac{\sqrt{\omega}}{\sqrt{2}\left[\overline{v^2}\omega^2\left(s + \bar{k}\Pi\right)^2 + 1\right]^{1/2}}\left[M_5 - \frac{1}{2}\in_\theta (M_5N_3 - M_6N_4)\right]$$

$$\mathrm{Im}(q_\theta) = \frac{\sqrt{\omega}}{\sqrt{2}\left[\overline{v^2}\omega^2\left(s + \bar{k}\Pi\right)^2 + 1\right]^{1/2}}\left[M_6 - \frac{1}{2}\in_\theta (M_6N_3 + M_5N_4)\right]$$

where $M_{5,6} = \left(\sqrt{N_5^2 + N_6^2} \pm N_5\right)^{1/2}$, $N_5 = (\tau s\bar{v}\omega^2 - 1)\bar{v}\omega(s + \bar{k}\Pi) + \omega(s\bar{v} + \tau)$

$$N_6 = (\tau s\bar{v}\omega^2 - 1) - \omega^2\bar{v}(s\bar{v} + \tau)(s + \bar{k}\Pi)$$

For large ω

$$M_5 \to \omega^{3/2}\sqrt{2\tau s}\bar{v}(s + \bar{k}\Pi)^{1/2}, \ \ M_6 = \frac{\sqrt{\omega}(\bar{v}s^2 + s\bar{v}\bar{k}\Pi + \tau\bar{k}\Pi)}{\sqrt{2\tau s}(s + \bar{k}\Pi)^{1/2}},$$

$$N_{3,4} \to 0, \quad Re(q_\theta) \approx \frac{\omega\sqrt{\tau s}}{(s + \bar{k}\Pi)^{1/2}}, \quad \mathrm{Im}(q_\theta) \approx \frac{(\bar{v}s^2 + s\bar{v}\bar{k}\Pi + \tau\bar{k}\Pi)}{2\bar{v}\sqrt{\tau s}(s + \bar{k}\Pi)^{3/2}}$$

Hence,

$$c_\theta \approx \frac{(s+\bar{k}\Pi)^{1/2}}{\sqrt{\tau s}} \text{ and } s_\theta \approx -\frac{(\bar{\nu}s^2 + s\bar{\nu}\bar{k}\Pi + \tau\bar{k}\Pi)}{2\bar{\nu}\sqrt{\tau s}(s+\bar{k}\Pi)^{3/2}}$$

for large ω.

Therefore, the thermal wave speed and the attenuation coefficient are independent of the visco-elastic parameters and depend only on the electromagnetic parameters and thermal relaxation time.

$$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_\theta s_\theta\right| = \frac{2\pi(\bar{\nu}s^2 + s\bar{\nu}\bar{k}\Pi + \tau\bar{k}\Pi)}{\omega\tau s\bar{\nu}(s+\bar{k}\Pi)}$$

For small ω, $N_3 \to -1, N_{4,5} \to 0, \ N_6 \to -1, M_{5,6} \to 1$.

$$Re(q_\theta) \approx \sqrt{\frac{\omega}{2}}\left(1+\frac{1}{2}\in_\theta\right), \quad \text{Im}(q_\theta) \approx \sqrt{\frac{\omega}{2}}\left(1+\frac{1}{2}\in_\theta\right).$$

Hence, $c_\theta \approx \sqrt{2\omega}\left(1-\frac{1}{2}\in_\theta\right)$ and $s_\theta \approx -\sqrt{\frac{\omega}{2}}\left(1+\frac{1}{2}\in_\theta\right)$

$$\frac{\Delta W}{W} = \left|\frac{4\pi}{\omega} c_\theta s_\theta\right| = 4\pi\left(1-\frac{1}{4}\in_\theta^2\right).$$

Thus, for the case of thermal waves, the specific energy loss is independent of any electro-magnetic and visco-elastic parameters at low frequency but influenced by the thermal coupling $\in_\theta$.

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